Prototyping Filter-Sum Beamformers for Sound Source Localization in Mobile Robotics

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***Abstract*— The work presented in this paper comes as a part of a project which aims at developing an auditory system for a mobile robot. It presents a sound source localization strategy which enables the sensing of signals within a direction of arrival and frequency domain of interest while rejecting other data. A rapid prototyping method is proposed to design filter-sum beamformers on the basis of convex optimization. This method is well-suited to robotics applications as it copes with real-time constraints and allows the localization of broadband signals such as human voice. Numerous simulation results are used to illustrate the reasoning.**

***Index Terms*— Microphone arrays, beamforming, sound-source localization, mobile robotics.**

I. Introduction

The LAAS-CNRS robotics group has been involved for many years in the development of navigation strategies for mobile robots. So far, most part of the developed methods have combined the use of vision, to perform localization or servoing tasks with respect to specific landmarks in the scene [2], with the use of ultrasonic sensors or 2D laser range finders to deal with obstacle avoidance [5] [1]. To complete the sensing capabilities of our robots, we have recently been working on the development of an auditory system. In order to be suited to relevant applications — such as voice localization and processing for robot navigation in interaction with humans— an acoustic sensor must be able to localize in real-time sound sources covering a wide frequency domain while rejecting other signals. In this sense, narrowband methods appear to be of limited interest.

Many auditory sytems developed for robotics applications are biologically inspired. They localize sound sources over a prescribed frequency bandwidth through the difference in phase (IPD) and in intensity level (IID) between two microphones, both of which can be computed from the *head related transfer function* (HRTF) [4] [9]. However, the determination of the HRTF requires precise measurements in an anechoic room, so that the application of such localization techniques to mobile robotics, in an evolutive environment, turns out to be limited. A geometrical method for extracting information on source location without using HRTFs was proposed in [15] and improved in [14]. To remove the front-back confusion occuring in most strategies using only two microphones, an auditory system made of three sensors was used to drive a mobile robot in [8].

However, even though the development of ear-like systems is rather challenging today, especially for humanoid robots, the use of microphone arrays including a larger number of sensors can increase the resolution of the localization procedure and its robustness with respect to noise. Furthermore, sensor arrays design has been a long-time research topic in acoustics, whose application to robotics has been proved efficient by recent works. A sound source localization method, based on *time delay of arrival* (TDOA), was proposed in [20] by using an array of eight microphones located on a rectangular prism. The system robustness was later improved by using a frequency-domain beamformer enhanced by a stochastic post-processing [19]. Application of microphone arrays to talker localization and speech recognition was proposed in [7].

These results have motivated our choice for developing the auditory system of our wheeled robot on the basis of a linear microphone array. We present in this paper a rapid prototyping method which allows to steer the acoustic array response so as to make it sensitive to signals within a direction of arrival and frequency domain of interest while rejecting other data. This method, inspired from classical beamforming techniques, is based on convex optimization.

II. Towards a beamforming approach to the localization of wideband acoustic signals

A. Acoustic waves propagation

a) Generalities: Under slight simplifying assumptions, the sound pressure field $x(\mathbf{r},t)$ at space location $\mathbf{r}$ and time t can be considered to be governed by a wave equation similar to the one used in electromagnetism [3] [13]. Two monochromatic solutions are of interest: the *near-field* solution — with spherical wavefront— and the *planar* —or *far-field*— solution given by $x(\mathbf{r},t) = Ae^{j(\omega t - \mathbf{k}.\mathbf{r})}$ with $\omega = 2\pi f$ and f the *temporal frequency* of the propagating monochromatic wave. Similarly, $\mathbf{k} = k\mathbf{u}$ is the *wavenumber vector*, with $\lambda = \frac{c}{f}$ the *wavelength*, $k = \frac{2\pi}{\lambda}$ the *wavenumber*, and $\mathbf{u}$ the unit vector supporting the direction of propagation of the wave. Following [13], when a spatially linear sensor is used, a sufficient condition for the far-field hypothesis to hold is the inequality $|\mathbf{r}| > \frac{2L^2}{\lambda}$ with L the dimension of the sensor.

b) Audio waveforms in robotics applications: This paper considers any robotics application in which the informatory acoustic wave is unique and far-field, propagating at the roughly constant speed $c \approx 340\,\mathrm{m.s}^{-1}$. Though the used microphones can only measure pressures oscillating at frequencies between 100 Hz and about 10 kHz, the ratio of the highest to the lowest frequency remains high so that a wideband wave model is more appropriate. Moreover, the considered indoor environments are expected to be reverberant and prone to additive low-frequency parasites generated by wheeled and tracked vehicles.

B. Array signal processing methods for source localization

c) Basics: The information is synchronously sampled at passive omnidirectional nearly identical microphones whose characteristics and locations are known. A collaborative processing is then performed. No *a priori* statistical assumption is made on the informatory wave nor on the environment noise. The proposed system can be run in real-time with nice computational properties (cf. § IV).

d) A direct method: steering the array pattern: The global response of the microphone array, or *array pattern*, is a function $P(\mathbf{k}, f)$ of the frequency and direction of the incoming wave. It can be tuned up to a suitable design through *beamforming*. For instance, spatial filtering can be designed so as to enhance the signal coming from a desired spatial direction while reducing it from the others. A direct method for acoustic source localization immediatly follows. It consists in making successive hypotheses on each potential source location. For each assumption, a steered beamformer —the characteristics of which have been determined offline— combines the microphones signals in order to maximize the output energy at the postulated direction. Among all these steered directions, the energy is then expected to be maximal at the actual source *direction of arrival* (DOA). This approach, which underlies the work presented here, can also be used in the near-field case [17].

C. Fundamentals of beamforming

e) Mathematical formulation: In the following, $.^T$ terms the non-Hermitian transpose operator. As shown in Figure 1, a rectilinear array is used, in which the i^{th} sensor is at location d_i on the x-axis of the world frame. Under the far-field assumption, only the wave DOA (θ, ϕ) is available. If the N array elements have the same response $a(\theta, \phi, f)$ and are combined through gains $w_i(f)$, then it can be shown that the whole antenna horizontal pattern for $\phi = \frac{\pi}{2}$ is [13] :

$$P(\theta, f) = \sum_{i=1}^{N} w_i(f) a(\theta, \frac{\pi}{2}, f) e^{j\frac{2\pi}{\lambda} d_i \sin\theta} = W(f)^T V(\theta, f) \qquad (1)$$

where $V(\theta, f) = a(\theta, \frac{\pi}{2}, f)(e^{j\frac{2\pi}{\lambda} d_1 \sin\theta}, \ldots, e^{j\frac{2\pi}{\lambda} d_N \sin\theta})^T$ is the *steering vector* and $W(f) = (w_1(f), \ldots, w_N(f))^T$ is the gains column vector. The case of omnidirectional microphones implies that $a(\theta, \phi, f) = a(f)$. From now on, without loss of generality, $a(f) = 1$ will be assumed.

A classification of beamforming techniques is shown in [10] [21]. Some of these propose a stochastic interpretation assuming some characteristics of the emitting source and of the environment noise. For instance, the squared norm $|W|^2$ of the gains column vector is related to the amplification by the beamformer of a spatial white-noise. It is thus termed *white-noise gain* and should remain into admissible limits.

f) Delay-sum beamforming for narrowband signals: Narrowband signals have a well-defined nominal wavelength $\lambda = \lambda_0$, so time-delays between sensors due to propagation can be compensated by simple phase shifts in order to make the array pattern most sensitive to some incoming direction. When identical sensors compose the array, the conventional or Bartlett beamformer selects $W = \frac{1}{N}(e^{-j\frac{2\pi}{\lambda_0} d_1 \sin\theta_0}, \ldots, e^{-j\frac{2\pi}{\lambda_0} d_N \sin\theta_0})^T$, which leads to a peaked pattern at the DOA θ_0. Other arguments can be selected, *e.g.* the celebrated design of Dolph or the minimum-variance Capon's beamformer [10]. Whatever the design, it must be kept in mind that a *spatial Shannon sampling theorem* must be satisfied, relating the array length and λ_0.

g) Filter-sum beamforming for broadband signals: To make the array sensitive to one spatial DOA along the whole range of frequencies, time delays must be exploited as opposed to phase differences in the narrowband case. Indeed, as shown in Figures 6(a)–7(a) a narrowband Bartlett beamformer tuned to the DOA $\theta_0 = 10°$ at $f_0 = 2\,\mathrm{kHz}$ can be sensitive to significantly different DOA as soon as the wave frequency contents is not limited to a close neighborhood of f_0; a second main lobe can be noticed in Figure 7(a) at high frequencies because of the aliasing induced by the nonsatisfaction of the spatial Shannon theorem. The selection of frequency-varying weights $W(f) = \frac{1}{N}(e^{-j\frac{2\pi f}{c} d_1 \sin\theta_0}, \ldots, e^{-j\frac{2\pi f}{c} d_N \sin\theta_0})^T$ makes the array sensitive to any wave incoming at θ_0 whatever its frequency, see Figures 6(b)–7(b). Though the array is uniformly sensitive to a $10°$ DOA, spatial aliasing cannot be avoided. Besides, the main lobe width is important at low frequency, endowing the

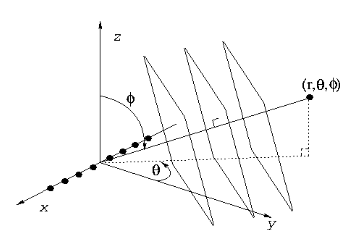

Fig. 1. Linear array in the far-field case

localization with a poor resolution. Generally, beamforming techniques for broadband signals use a K-order FIR associated to each of the N microphones. Consequently, the array pattern can be written as [21] [16]:

$$P(\theta, f) = \sum_{i=1}^{N} \sum_{k=1}^{K} w_{i,k} \; e^{-j2\pi f(k-1)T_e} \; e^{j\frac{2\pi f}{c} d_i \sin\theta} \qquad (2)$$

where $w_{i,j}$ is the j^{th} FIR coefficient of the i^{th} microphone. Similarly to (1), the matrix formulation of this equation [1] is:

$$P(\theta, f) = W^T V(\theta, f) \qquad (3)$$

with $V(\theta, f) = V_{Array}(\theta, f) \otimes V_{FIR}(f)$,

$$W = \begin{pmatrix} w_{1,1} \\ w_{1,2} \\ \vdots \\ w_{i,k} \\ \vdots \\ w_{N,K} \end{pmatrix}, \; V_{FIR}(f) = \begin{pmatrix} 1 \\ e^{-j2\pi f T_e} \\ e^{-j2\pi f 2T_e} \\ \vdots \\ e^{-j2\pi f(K-1)T_e} \end{pmatrix}$$

$$\text{and } V_{Array(\theta,f)} = \begin{pmatrix} e^{j2\pi d_1 \frac{f}{c}} \sin\theta \\ e^{j2\pi d_2 \frac{f}{c}} \sin\theta \\ \vdots \\ e^{j2\pi d_N \frac{f}{c}} \sin\theta \end{pmatrix}.$$

h) A convex optimization framework to the design of beamformers: When using a linear microphone array with constant intersample distance d, delay-sum beamforming shows similarities with FIR temporal filtering. So, methods for FIR filters design can be used for delay-sum beamformers design, provided the distance d replaces the sampling period T_e. Many FIR filters design methods rely on convex optimization techniques [23]. Thanks to the flexibility of this framework, various design strategies can be considered while taking into account possibly conflicting constraints. These problems are then numerically solved —often in polynomial time— as the convergence to the global minimum is guaranteed as soon as the set of constraints is feasible. Similarly, convex optimization methods have been proposed to the design of array pattern synthesis (APS). In [22] a new method is proposed to make the magnitude response of a narrowband unevenly spaced array fit a reference pattern. Other approaches concern broadband arrays. In [11], a convex formulation of the APS problem is proposed in the case of spherical sound waves under equality constraints on the desired pattern. In the same way, [16] takes into account power constraints over several frequency ranges.

[1] $\otimes$ represents the Kronecker product : given two matrices X and Y, whose respective dimensions are $p \times q$ and $t \times l$, the Kronecker product $X \otimes Y$ is the matrix Z of dimension $pt \times ql$ formed by the pq submatrices $Z_{i,j} = X_{i,j}Y$, for $(i, j) \in \{1, \ldots, p\} \times \{1, \ldots, q\}$.

III. A BROADBAND ARRAY PATTERN SYNTHESIS USING CONVEX OPTIMIZATION

Handling microphones phase uncertainties during the design of acoustic antennas is fundamental for real applications. This reason has motivated our wish to extend the approach proposed in [22] to the case of broadband arrays. Denoting by $P_d(\theta, f)$ the desired array pattern, a synthesis strategy can be stated as the following optimization problem on the matrix W made of the $N \times K$ FIR taps:

$$\begin{array}{ll} \text{minimize} & \varepsilon \\ \text{subject to:} & |W^T V(\theta, f) - P_d(\theta, f)|^2 \leq \varepsilon, \; \forall (\theta, f) \in \Theta \times F, \end{array} \qquad (4)$$

where Θ and F term the sets of selected DOA and frequencies. Compared to [22], the temporal frequency dimension has just been added to the problem formulation. Because of the contraints it involves, (4) is a particular convex optimization problem, called second-order cone program (SOCP). Such a problem can be efficiently solved by specialized solvers such as SeDuMi, SDPT3, MOSEK, etc. The used solver SDPT3 (v3.2) [18] is coupled with YALMIP [12] and runs under MATLAB.

The auditory system is constituted by a linear array of $N = 8$ equispaced microphones. For the optimization process, the frequency set F is sampled at $100Hz$, and the angle set Θ is sampled at $1°$. The acquisition sampling frequency is fixed to $f_e = 8kHz$. The desired pattern which has been designed to detect the meaningful signal is represented in Figure 2. There, θ_c is the main lobe center which corresponds to the listening direction, θ_p defines the bandwidth in θ, and θ_s defines the main lobe width. Such a pattern is then defined for each $\theta_c \in \Theta$.

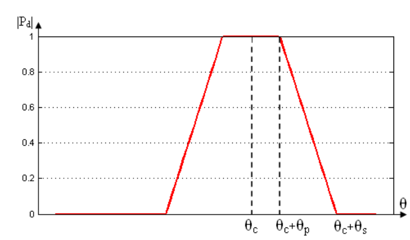

Fig. 2. Desired array pattern parameterized by θ_c, θ_s and θ_p.

A. Spatial filtering analysis

In this section, we present a first set of simulation results which has been obtained by duplicating the reference pattern along the bandwidth, and we discuss the influence of the different parameters.

a) Frequency response: Consider the case when the objective is to listen to the direction $\theta_c = 0°$ while keeping a *constant* main lobe width for each frequency $f \in F = [500Hz; 2000Hz]$. For this example, we fix $\theta_p = 5°$ and $\theta_s = 20°$. The solution to the optimization problem (4) is described in Figure 3 for various frequencies. The response

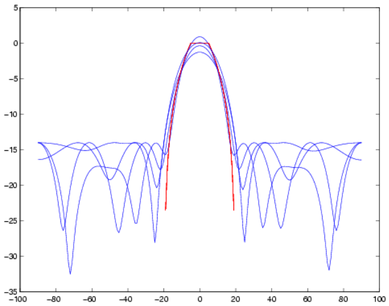

Fig. 3. Optimized wideband array pattern in dB for various frequencies with a 15-order FIR (the desired pattern is plotted in red).

fits well with the desired pattern and the width of the main lobe remains almost constant over the different frequencies. However, oscillations occur in the main lobe, inducing more attenuation at low frequencies. In this example, the level difference in the main lobe is about $3dB$, while the spatial filtering remains efficient: the side-lobe level is about $-12dB$. Other results obtained for $\theta_c = 10^\circ$ are shown in Figure 6(c) while Figures 6(a) and 6(b) represent the responses obtained with classical beamforming approaches. The conservation of the main lobe direction over the frequency set and the improvement of the resolution at low frequencies is clearly confirmed.

b) Influence of the FIR filter order K: The increase of the FIR filter order K induces a better spatial rejection. But, as a result, the larger the filter order, the larger the weight power, see Figure 4. The progression of the weight power

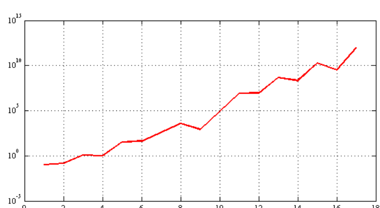

Fig. 4. Weight power W^TW *vs* K for $\theta_s = 20^o$.

W^TW is roughly exponential. As previously explained, this weight power is the white-noise gain. So, to guarantee good performances against noise, it is important to bound this value. The first idea is to reduce the order of each FIR filter. However, decreasing the order of the FIR comes to reduce the number of degrees of freedom, and therefore to increase the difference between the response and the desired pattern: the side-lobes level and the main lobe width for the lowest frequencies increase. In the sequel, we consider the influence of the other parameters for $K = 17$.

c) Influence of the white-noise gain upper bound: The proposed formulation allows to consider additional constraints easily. For instance, let δ be an upper bound on the white-noise gain. By adding the new SOCP constraint $W^TW \leq \delta$, it is possible, for a given FIR order, to limit the weight power. As shown in Figure 5-left, high constraints on the weight power induce an increase of the side-lobe levels and reduce the performances along frequencies.

d) Influence of the main lobe width θ_s: We now consider the influence of the width parameter θ_s for $K = 17$ and $\delta = 10$. To obtain a good rejection level for severe weight power constraints, it is possible to enlarge the main lobe width as shown in Figure 5-right. So, for a large main lobe, the side lobe level remains under 20 dB, but the spatial filtering is less efficient.

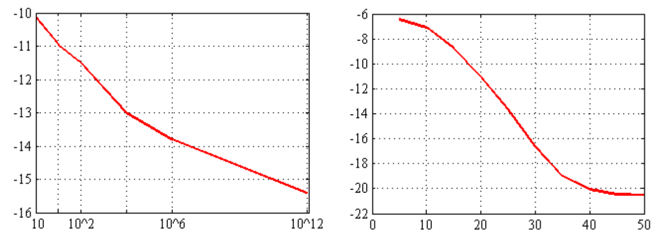

Fig. 5. Side-lobe level in dB (for a 17 order FIR) vs : - white noise gain upper bound δ with $\theta_s = 20^\circ$ (left) - main lobe width θ_s with $W^TW \leq 10$ (right).

e) Conclusion: To conclude on this first design, it appears necessary to find a compromise between spatial filtering efficiency (side-lobe level and main lobe width) and weight power value (white-noise gain). The performance turns out to be dependent on the array dimensions and on the number of microphones. For this reason, classical beamforming techniques turn out to be weakly efficient to localize low frequency signals.

B. Spatial and frequency filtering analysis

So far, we have considered that the frequency band of the sound source was identical to the whole frequency domain F upon which the optimization process is based. However, in the case of spurious signals containing at least one frequency greater than the upper bound of F, the above pattern can lead to a wrong detection. Typically, the DOA of the noisy signal can be detected instead of the DOA of the sound source. To cope with this problem, we show in this section that the flexibility of our approach allows to design an array performing both spatial and frequency filtering. To illustrate this point, we show in the sequel that the minimization procedure can be extended to the case when the frequency domain F is larger than the frequency band of the source signal. For the same source signal as before, we have considered that the frequency domain F is [500Hz;4kHz]. As a consequence, we want all frequencies upper than 2kHz to be filtered by the beamformer. The result is shown in Figure 7.

Let us analyze now our approach. Contrarily to classical beamforming techniques, the main lobe width is maintained quasi-constant for the frequencies of interest, as in the previous simulations. The same remarks as before can be done concerning the oscillations and the peak value of the main-lobe. The ability to reject noisy frequencies greater than 2kHz allows to avoid the spatial aliasing effect. As a result, this approach, based on the precise knowledge of the array pattern, constitutes a way to avoid a large part of false sound source detections.

IV. Interest for robotic applications

Our robotic auditory system is currently under development. It involves a linear array of eight uniformly spaced GRAS 40PQ microphones. The acquisition board is an 8-channel Audio-AUPM from Bittware equipped with Dual 21065L Analog Devices DSPs. Considering the DSP characteristics, the localization process is expected to run at the minimum frequency of 15Hz. This would cope with the real-time constraints of mobile robotics applications. On the other hand, the ability to detect broadband signals is well-adapted to talker localization and speech recognition which are of great interest for human-robot interaction. Furthermore, the proposed approach constitutes a rapid array prototyping technique which allows to design the FIR taps according to the application. More precisely, the method can be developed with the MATLAB/SIMULINK software from which the DSP developer interface allows to generate automatically the DSP code. In this way, just after the off-line determination of the FIR taps, the array performance can be immediately evaluated on-line.

V. Conclusion

A method has been presented enabling the rapid prototyping of an acoustic antenna for robotics applications. The results, which have been so far illustrated by simulations, show the flexibility of the approach for a wide range of problems requiring the localization of broadband signals. We are currently working on the technical development of the auditory system. In parallel, our effort is turned to the following points: designing patterns only defined in magnitude, endowing the system with robustness properties with respect to microphone phase uncertainties, and improving the mathematical conditioning of the optimization problem. On the basis of these results, we plan to use acoustic cues in multi-sensor based control as well as in audio-visual tracking for human robot interaction, in the vein of [6].

VI. Acknowledgments

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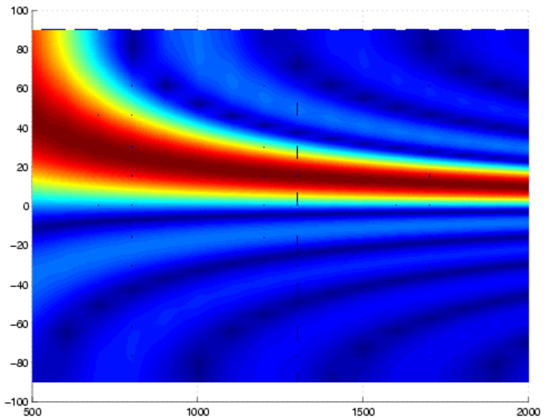
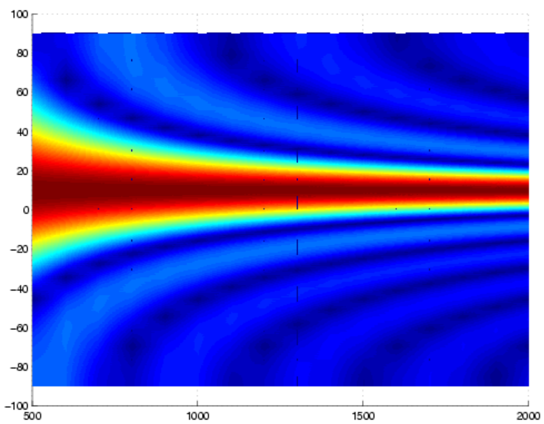
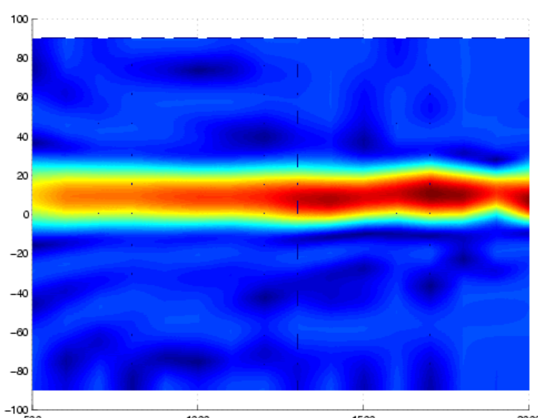


(a) Delay-Sum Beamforming

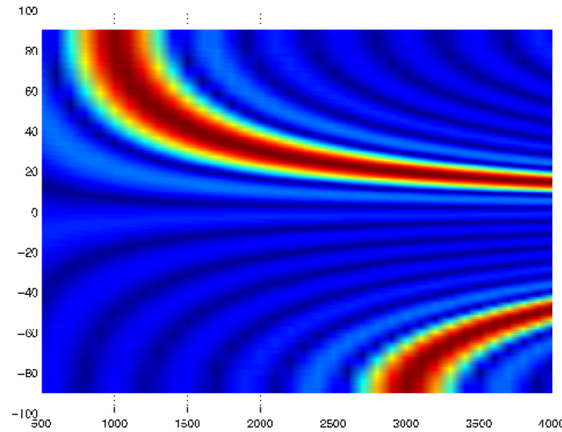


(b) Filter-Sum Beamforming

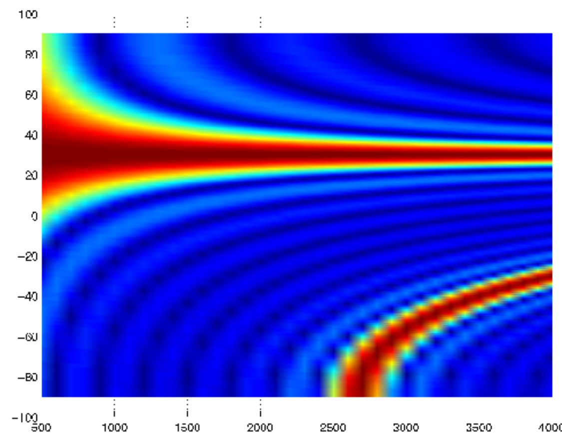


(c) Optimized Wideband Array Pattern

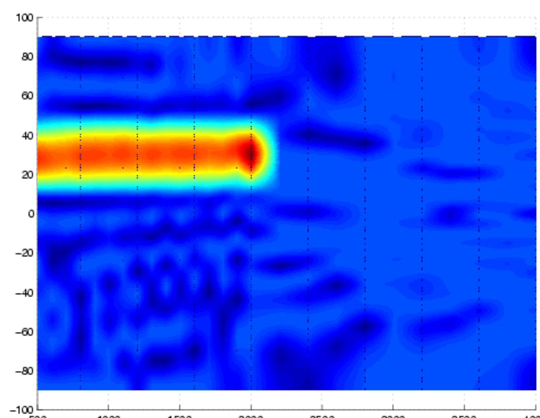
Fig. 6. Comparison between classical beamforming techniques and our approach for $\theta_c = 10^o$, $\theta_p = 5^o$, $\theta_s = 20^o$ and a 15 order FIR. The Delay-Sum beamformer has been designed to be sensitive to the direction 10° at 2kHz. The Filter-Sum has been designed to be sensitive to the direction 10°. Spacing between microphones is adjusted to $d = \lambda_{min}/2$ with $\lambda_{min} = 2kHz/c$. Vertical axis is θ in degrees, horizontal axis is frequency in Hz. Hot colors represent peaked values.



(a) Delay-Sum Beamforming



(b) Filter-Sum Beamforming



(c) Optimized Wideband Array Pattern

Fig. 7. Comparison between classical beamforming techniques and our approach for $\theta_c = 30^o$, $\theta_p = 5^o$, $\theta_s = 20^o$, a 35 order FIR. The Delay-Sum beamformer has been designed to be sensitive to the direction 30° at 2kHz. The Filter-Sum has been designed to be sensitive to the direction 30°. Spacing between microphones is adjusted to $d = \lambda_{min}/2$ with $\lambda_{min} = 2kHz/c$. Vertical axis is θ in degrees, horizontal axis is frequency in Hz. Hot colors represent peaked values.